Spin dynamics of two-dimensional electrons with Rashba spin-orbit coupling and electron-electron interactions

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We study the spin dynamics of two-dimensional electron gases with Rashba spin-orbit coupling by taking account of electron-electron interactions. The diffusion equations for charge and spin densities are derived by making use of the path-integral approach and the quasiclassical Green's function. Analyzing the effect of the interactions, we show that the spin-relaxation time can be enhanced by the electron-electron interaction in the ballistic regime.

I. INTRODUCTION

Spin-based electronics[1] or spintronics[2] has been an active research area in the past decade. The effort for effectively manipulating electron spin by means of an applied electric field[3–5] is an important issue there. The system with spin-orbit (SO) couplings makes those efforts possible and thus brings great interest from both academic and practical aspects recently. Thus, it is essential to study the spin relaxation for further development of spintronics.

There are four main mechanisms of spin relaxation in semiconductor systems.[2,6–10] In the Elliott-Yafet mechanism, the spin-orbit coupling induces a mixing of wave functions for valence-band and conduction-band states. The mixing that results in the spin relaxation of electrons is due to the scattering by impurities or phonons. The Elliott-Yafet mechanism operates in semiconductors with and without a center of inversion symmetry, while it is most prominent in the centrosymmetric ones (such as silicon). The Bir-Aronov-Pikus mechanism is applicable for *p*-doped semiconductors in which the electron-spin flipping is induced by exchange interaction with holes. The hyperfine interaction provides another important mechanism[11] for ensemble spin dephasing and single spin decoherence of localized electrons. The D'yakonov-Perel mechanism depicts that electrons can feel an effective random magnetic field arising from the spin-orbit coupling in systems with inversion asymmetry such that spin relaxation occurs. This mechanics can interpret the spin dephasing in crystals without inversion center and is particularly applicable for *n*-type samples. For two-dimensional *n*-type semiconductor systems without inversion symmetry, the D'yakonov-Perel mechanism is believed to be most important in wide ranges of carrier temperature and concentration. Under certain conditions, the Elliott-Yafet mechanism may affect spin dynamics of two-dimensional electrons in these systems. The Bir-Aronov-Pikus mechanism is important for *p*-type semiconductor systems and the hyperfine-interaction mechanism dominates for localized electrons.

Most studies of spin relaxation in semiconductors have focused on impurity(somewhat less phonon)-mediated spin flips while neglecting the effect of electron-electron interactions for a long time. It has been noticed recently that electron-electron interactions play certain role in spin relaxation and dephasing in semiconductor systems. The electron-electron interaction is known to play a crucial role in determining the transport and thermodynamic properties near the metal-insulator transition in two-dimensional electron systems,[12] which is suspected to affect the spin relaxation for the spin susceptibility behaving critically when the metal-insulator transition occur.[13–15] There are several experimental and theoretical studies on the effect of electron-electron interactions on spin relaxation. The electron-electron scattering results in additional momentum relaxation which induces spin dephasing of electrons through the motional narrowing of the D'yakonov-Perel type[16] as measured in *n*-GaAs/AlGaAs quantum wells.[17,18] The electron-electron scattering effect on the spin dephasing has been considered[19] in a magnetic field, and a momentum-dependent effective random magnetic field induced by the electron-electron exchange interaction can lead to spin dephasing of electrons.[20–22] It is also observed that the spin relaxation caused by the D'yakonov-Perel mechanism gives considerably different rates depending on the technique employed.[23]

However, as we are aware, the explicit form of the diffusion equation for two-dimensional electron gases (2DEGs) with spin-orbit couplings has not been derived by taking account of electron-electron interactions. It is thus obligatory to develop the explicit form of the diffusion equation to study the spin dynamics for 2DEGs with spin-orbit couplings as well as electron-electron interactions. In this paper, we focus our attention on the D'yakonov-Perel spin-relaxation mechanism. We investigate the spin dynamics of electrons in two-dimensional *n*-type semiconductor systems with electron-electron interactions and Rashba spin-orbit coupling.

The paper is organized as follows. In Sec. II, we take account of the electron-electron interaction for the 2DEGs with the Rashba spin-orbit coupling. Applying the path-integral formulation, we decouple the interaction in terms of an auxiliary Bose field. In Sec. III, we employ the quasiclassical Green's function to investigate the spin dynamics of electrons. In Sec. IV, the diffusion equations for spin and charge densities as well as the explicit expression of spin-relaxation time are derived. A summary is given in Sec. V and some complicated formulae are given in the Appendix.

II. AUXILIARY FIELDS DESCRIBING THE ELECTRON-ELECTRON INTERACTION

Taking the electron-electron interaction into account, we study the spin dynamics of electrons in two-dimensional systems with structure inversion asymmetry. As the Fourier transform of the Coulomb repulsion between electrons reads $V(\mathbf{q}) = 2\pi e^2/|\mathbf{q}|$, the Hamiltonian of such a system is given by

$$\hat{H} = \int \left\{ \sum_{\lambda,\lambda'} \hat{\psi}^{\lambda\dagger}(r) \left[\left(-\frac{\hbar^2}{2m}\nabla^2 + U(r) - \mu \right) \delta_{\lambda,\lambda'} \right.\right.$$
$$\left.\left. + \mathbf{b} \cdot \vec{\sigma}_{\lambda\lambda'} \right] \hat{\psi}^{\lambda'}(r) \right\} d^2r + \frac{1}{A} \sum_{\mathbf{q}\neq 0} \frac{\pi e^2}{|\mathbf{q}|} \hat{\rho}(\mathbf{q})\hat{\rho}(-\mathbf{q}), \quad (1)$$

where $\hat{\psi}^{\lambda\dagger}(r)$ and $\hat{\psi}^{\lambda}(r)$ represent the field operators with $\lambda = \uparrow, \downarrow$ labeling the spin state of the electron, $\hat{\rho}(\mathbf{q})$ represents the Fourier transform of the density operator $\hat{\rho}(\mathbf{r}) = \sum_\lambda \hat{\psi}^{\lambda\dagger}(r)\hat{\psi}^{\lambda}(r)$, and $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ represents the Pauli matrices in spin space, $U(r)$ represents a random disorder potential, and μ represents the chemical potential. The other notions in Eq. (1) are $A = L^2$ with L referring to the size of the sample and $\mathbf{b} = \alpha\mathbf{p} \times \mathbf{e}_z$ with α referring to the Rashba spin-orbit coupling strength. In holonomy representation (or the so-called coherent-state representation), the Green's function can be expressed as a functional integral over the Grassmann fields ψ^λ and $\bar{\psi}^\lambda$ that reflect the fermionic nature of electrons,

$$G_{\lambda\lambda'}(r,t;r',t') = \langle \psi^\lambda(r,t)\bar{\psi}^{\lambda'}(r',t') \rangle$$
$$= \frac{\int D\bar{\psi}D\psi\,\psi^\lambda(r,t)\bar{\psi}^{\lambda'}(r',t')e^{-iS[\psi,\bar{\psi}]}}{\int D\psi D\bar{\psi}\,e^{-iS[\psi,\bar{\psi}]}}. \quad (2)$$

Here we adopted the unit $\hbar = 1$ and the simplified notation $D\psi = D\psi^\uparrow D\psi^\downarrow$. The action $S[\psi,\bar{\psi}]$ in the above equation is given by

$$S[\psi,\bar{\psi}] = \int dt \left\{ \int d^2r \sum_{\lambda\lambda'} \bar{\psi}^\lambda(\mathbf{r},t)W_{\lambda\lambda'}\psi^{\lambda'}(\mathbf{r},t) \right.$$
$$\left. + \frac{1}{A}\sum_{\mathbf{q}\neq 0}\frac{\pi e^2}{|\mathbf{q}|}[\rho(\mathbf{q},t)\rho(-\mathbf{q},t)] \right\}, \quad (3)$$

where $W_{\lambda\lambda'} = [-i\partial/\partial t - \nabla_r^2/2m + U(r) - \mu]\delta_{\lambda\lambda'} + \mathbf{b}\cdot\vec{\sigma}_{\lambda\lambda'}$.

We divide the fermionic field $\psi^\lambda(\mathbf{r},t)$ into two components $\psi_1^\lambda(\mathbf{r},t)$ and $\psi_2^\lambda(\mathbf{r},t)$ which reside, respectively, on the upper and lower branches of the Keldysh time contour shown in Fig. 1. Hence, the second line of Eq. (3), which refers to the interaction part, can be written as $S_{\text{int}}[\psi_1,\bar{\psi}_1] - S_{\text{int}}[\psi_2,\bar{\psi}_2]$ with

$$S_{\text{int}}[\psi_i,\bar{\psi}_i] = \int dt \sum_{\mathbf{q}\neq 0}\frac{\pi e^2}{A|\mathbf{q}|}[\rho_i(\mathbf{q},t)\rho_i(-\mathbf{q},t)],$$

where $i = 1, 2$. With the help of two auxiliary bosonic fields $\tilde{\phi}_i(\mathbf{r},t)$, we can decouple those two terms relevant to
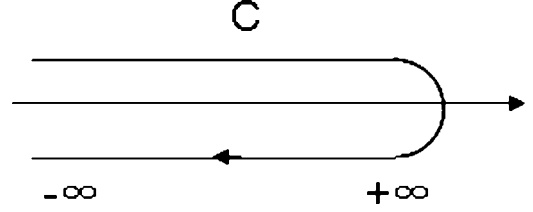

FIG. 1. The Keldysh contour.

electron-electron interactions via the Hubbard-Stratonovich transformation,[24] namely,

$$\exp\left[-i\int dt\sum_{\mathbf{q}\neq 0}\frac{\pi e^2}{A|\mathbf{q}|}\rho_i(\mathbf{q},t)\rho_i(-\mathbf{q},t) \right]$$
$$= \int D\tilde{\phi}_i(\mathbf{q},t)\exp\left[i\int dt\sum_{\mathbf{q}\neq 0}\frac{|\mathbf{q}|}{4\pi}\tilde{\phi}_i(\mathbf{q},t)\tilde{\phi}_i(-\mathbf{q},t) \right]$$
$$\times \exp\left[i\int dt\frac{e}{2\sqrt{A}}\sum_{\mathbf{q}\neq 0}\{\tilde{\phi}_i(\mathbf{q},t)\rho_i(-\mathbf{q},t) \right.$$
$$\left. + \rho_i(\mathbf{q},t)\tilde{\phi}_i(-\mathbf{q},t)\} \right].$$

Then we can write the Green's function as follows:

$$\hat{G}_{\lambda\lambda'}(r,t;r',t') = \frac{\int D\bar{\Psi}D\Psi D\Phi\,\Psi^\lambda(r,t)\bar{\Psi}^{\lambda'}(r',t')e^{-iS[\Psi,\bar{\Psi},\Phi]}}{\int D\Psi D\bar{\Psi}D\Phi\,e^{-iS[\Psi,\bar{\Psi},\Phi]}},$$
$$(4)$$

where the action in real space is given by

$$S[\Psi,\bar{\Psi},\Phi] = \int dt d^2r \left\{ \sum_{\lambda\lambda'}\bar{\Psi}^\lambda(\mathbf{r},t) \right.$$
$$\times [W_{\lambda\lambda'}\sigma_3 - e\tilde{\phi}_i\tilde{\gamma}_i\delta_{\lambda\lambda'}]\Psi^{\lambda'}(\mathbf{r},t) \Big\}$$
$$+ \int dt \int d^2r d^2r' \left\{ \Phi^T(\mathbf{r},t)\frac{-e^2}{2} \right.$$
$$\left. \times V_0^{-1}(\mathbf{r}-\mathbf{r}')\sigma_3\Phi(\mathbf{r}',t) \right\}, \quad (5)$$

in which the Pauli matrix $\sigma_3 = \text{diag}(1,-1)$ is defined on the Keldysh space, and V_0^{-1} is defined via the following relation:

$$\int d^2r_1 V_0(\mathbf{r}-\mathbf{r}_1)V_0^{-1}(\mathbf{r}_1-\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}').$$

The other notions appeared in Eq. (5) are fermionic doublet Ψ, bosonic doublet Φ, and vertex matrices $\tilde{\gamma}_i$. They are defined as

$$\Psi^\lambda = \begin{pmatrix} \psi_1^\lambda \\ \psi_2^\lambda \end{pmatrix}, \quad \Phi = \begin{pmatrix} \tilde{\phi}_1 \\ \tilde{\phi}_2 \end{pmatrix},$$

$$\widetilde{\gamma}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \widetilde{\gamma}_2 = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}.$$

For calculation convenience, one can introduce a partition function for the coupling between the fermionic and bosonic doublets,

$$Z[\Phi] = \langle T_C e^{-iS_R[\Phi,\Psi]} \rangle_\Psi,$$

$$S_R[\Phi,\Psi] = \int dt d^2 r \left\{ \sum_\lambda \bar{\Psi}^\lambda (-e \widetilde{\phi}_i \widetilde{\gamma}_i) \Psi^\lambda \right\},$$

where T_C stands for time ordering along the contour C and $\langle \cdots \rangle_\Psi$ means functional integration over Ψ field with the action

$$S[\Psi] = \int dt d^2 r \left\{ \sum_{\lambda\lambda'} W_{\lambda\lambda'} \bar{\Psi}^\lambda(\mathbf{r},t) \sigma_3 \Psi^{\lambda'}(\mathbf{r},t) \right\}.$$

Then the Green's function in Eq. (4) can be formally expressed as a functional integration over the bosonic fields,

$$\hat{G}_{\lambda\lambda'}(\mathbf{r},t;\mathbf{r}',t') = \mathbb{N} \int D\Phi \hat{G}_{\lambda\lambda'}(\mathbf{r},t;\mathbf{r}',t'|\Phi) \exp\{-iS_e[\Phi]\}, \tag{6}$$

where the normalization coefficient is denoted by $\mathbb{N}$ and the action $S_e[\Phi]$ is defined by

$$S_e[\Phi] = i \ln Z[\Phi] + \int dt d^2 r d^2 r'$$

$$\times \left\{ \Phi^T(\mathbf{r},t) \frac{-e^2}{2} V_0^{-1}(\mathbf{r}-\mathbf{r}') \sigma_3 \Phi(\mathbf{r}',t) \right\}, \tag{7}$$

and the kernel $\hat{G}(\mathbf{r},t;\mathbf{r}',t'|\Phi)$ is given by

$$\hat{G}_{\lambda\lambda'}(\mathbf{r},t;\mathbf{r}',t'|\Phi) = \frac{1}{Z[\Phi]} \langle T_C \Psi^\lambda(\mathbf{r},t) \bar{\Psi}^{\lambda'}(\mathbf{r}',t') e^{-iS_R[\Phi,\Psi]} \rangle_\Psi. \tag{8}$$

We can average the Green's function $\hat{G}_{\lambda\lambda'}(\mathbf{r},t;\mathbf{r}',t')$ over disorder as follows:[25]

$$\langle \hat{G}_{\lambda\lambda'}(\mathbf{r},t;\mathbf{r}',t') \rangle_{\mathrm{dis}} = \mathbb{N} \int D\Phi \langle \hat{G}_{\lambda\lambda'}(\mathbf{r},t;\mathbf{r}',t'|\Phi) \rangle_{\mathrm{dis}}$$

$$\times \exp\{-i\langle S_e[\Phi] \rangle_{\mathrm{dis}}\}, \tag{9}$$

where $\langle \cdots \rangle_{\mathrm{dis}}$ refers to the average over disorder. The random disorder potential $U(r)$ is assumed to be characterized by a correlation function,

$$\langle U(r)U(r') \rangle_{\mathrm{dis}} = \frac{1}{2\pi\nu\tau} \delta(r-r'),$$

where $\nu = m/\pi\hbar^2$ stands for the density of states. The average of the Green's function over disorder introduces the elastic-scattering time τ which is relevant to the random disorder. We neglect correlations between the mesoscopic fluctuations of $\langle S_e[\Phi] \rangle_{\mathrm{dis}}$ and the fermionic operators in Eq. (8) so that the average of the Green's function $\hat{G}_{\lambda\lambda'}(\mathbf{r},t;\mathbf{r}',t'|\Phi)$ can

be separated from the bosonic action $S_e[\Phi]$. This approximation is valid since the mesoscopic fluctuation is smaller than average quantities.

After averaging over disorder, we rotate the Keldysh bases $\hat{G} \rightarrow L\sigma_3 \hat{G} L^\dagger$ through a unitary matrix L,

$$L = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix},$$

so that the Green's function takes the following shape:

$$\hat{G}(\mathbf{r},t;\mathbf{r}',t'|\Phi) = \begin{pmatrix} G^R(\mathbf{r},t;\mathbf{r}',t'|\Phi) & G^K(\mathbf{r},t;\mathbf{r}',t'|\Phi) \\ G^Z(\mathbf{r},t;\mathbf{r}',t'|\Phi) & G^A(\mathbf{r},t;\mathbf{r}',t'|\Phi) \end{pmatrix}. \tag{10}$$

Note that the Green's function $\hat{G}(\mathbf{r},t;\mathbf{r}',t'|\Phi)$ is a 2×2 matrix defined in the Keldysh space, of which the matrix entities are again 2×2 matrices defined in spin space.

The bosonic fields after rotation take the following two components $\phi_1 = \frac{e}{2}(\widetilde{\phi}_1 + \widetilde{\phi}_2)$ and $\phi_2 = \frac{e}{2}(\widetilde{\phi}_1 - \widetilde{\phi}_2)$ that reside on the upper and lower branches of the contour C, respectively. Then the corresponding vertex matrices turn to $\gamma_{1(2)} = L(\widetilde{\gamma}_1 \pm \widetilde{\gamma}_2)\sigma_3 L^\dagger$, namely,

$$\gamma_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \gamma_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

and the interaction term $\bar{\Psi}^\lambda(-e\widetilde{\phi}_i\widetilde{\gamma}_i)\Psi^\lambda$ becomes $-\bar{\Psi}^\lambda \left(\begin{smallmatrix} \phi_1 & \phi_2 \\ \phi_2 & \phi_1 \end{smallmatrix} \right) \Psi^\lambda$. As γ_1 and γ_2 constitute a representation of Z_2 group, the interaction can be regarded as the coupling between Fermi field and Z_2 Bose field.

These Bose fields define the following propagators:

$$D^K(\mathbf{r_1},\mathbf{r_2};t_1,t_2) = -2i\langle \phi_1(\mathbf{r_1},t_1)\phi_1(\mathbf{r_2},t_2) \rangle,$$

$$D^R(\mathbf{r_1},\mathbf{r_2};t_1,t_2) = -2i\langle \phi_1(\mathbf{r_1},t_1)\phi_2(\mathbf{r_2},t_2) \rangle,$$

$$D^A(\mathbf{r_1},\mathbf{r_2};t_1,t_2) = -2i\langle \phi_2(\mathbf{r_1},t_1)\phi_1(\mathbf{r_2},t_2) \rangle,$$

$$\langle \phi_2(\mathbf{r_1},t_1)\phi_2(\mathbf{r_2},t_2) \rangle = 0. \tag{11}$$

One can show that those propagators obey the Dyson equations in the saddle-point approximation, namely,

$$\hat{D}(x_1,x_2) = \hat{D}_0 + \int dx_3 dx_4 \hat{D}_0(x_1,x_3)\hat{\Pi}(x_3,x_4)\hat{D}(x_4,x_2), \tag{12}$$

with notations $x \equiv (\mathbf{r},t)$, $D_0^R(q) = D_0^A(q) = -2\pi e^2/|\mathbf{q}|$, and

$$\hat{D} = \begin{pmatrix} D^R & D^K \\ 0 & D^A \end{pmatrix}, \quad \hat{D}_0 = \begin{pmatrix} D_0^R & 0 \\ 0 & D_0^A \end{pmatrix}, \quad \hat{\Pi} = \begin{pmatrix} \Pi^R & \Pi^K \\ 0 & \Pi^A \end{pmatrix},$$

$$\Pi^R(x_1, x_2) = \Pi^A(x_2, x_1) = \frac{\delta \operatorname{Tr}_s G^K(\mathbf{r}_1; t_1, t_1 | \Phi)}{-2i \, \delta \phi_1(\mathbf{r}_2, t_2)},$$

$$\Pi^K(x_1, x_2) = \frac{\delta \operatorname{Tr}_s [G^K(\mathbf{r}_1; t_1, t_1 | \Phi) + G^Z(\mathbf{r}_1; t_1, t_1 | \Phi)]}{-2i \, \delta \phi_2(\mathbf{r}_2, t_2)}, \tag{13}$$

where Tr_s stands for the trace in the spin space.

III. KINETIC EQUATION

In Sec. II, the electron-electron interaction has been decoupled with the help of auxiliary bosonic fields $\phi_{1(2)}$. This means that the influence of the interaction can be described by a Z_2 Bose field,

$$\hat{\varphi}(\mathbf{r}, t) = \begin{pmatrix} \phi_1(\mathbf{r}, t) & \phi_2(\mathbf{r}, t) \\ \phi_2(\mathbf{r}, t) & \phi_1(\mathbf{r}, t) \end{pmatrix}.$$

Now we are able to apply the quasiclassical Green's function[26–28] approach to study the spin dynamics. We derive the Eilenberger equation from the right-hand and left-hand Dyson equations obeyed by the Green's function $\hat{G}(\mathbf{r}, t; \mathbf{r}', t' | \Phi)$ in Eq. (10),

$$\tilde{\partial}_t \hat{g} + \mathbf{v}_F \cdot \vec{\nabla} \hat{g} + i[\mathbf{b} \cdot \vec{\sigma}, \hat{g}] = \frac{\hat{g} \langle \hat{g} \rangle_{\mathbf{n}} - \langle \hat{g} \rangle_{\mathbf{n}} \hat{g}}{2\tau}, \tag{14}$$

where $\mathbf{v}_F$ denotes the Fermi velocity, τ is the elastic-scattering time arising from the adoption of the standard self-consistent Born approximation; $\langle \cdots \rangle_{\mathbf{n}}$ means taking average over the direction of the electron momentum $\mathbf{n} = \mathbf{p} / |\mathbf{p}| \equiv (\cos\theta, \sin\theta)$, and the covariant derivative is defined by

$$\tilde{\partial}_t \hat{g} = \partial_{t_1} \hat{g} + \partial_{t_2} \hat{g} + i \hat{\varphi}(\mathbf{r}, t_1) \hat{g} - i \hat{g} \hat{\varphi}(\mathbf{r}, t_2). \tag{15}$$

The quasiclassical Green's function in Keldysh and spin spaces,

$$\hat{g} = \begin{pmatrix} g^R & g^K \\ g^Z & g^A \end{pmatrix}, \tag{16}$$

can be derived by integrating the Fourier transform of the Green's function in Eq. (10) over energy variables, i.e.,

$$\hat{g}(t_1, t_2; \mathbf{n}, \mathbf{r}) = \frac{i}{\pi} \int d\xi \, \hat{G}(t_1, t_2; \mathbf{p}, \mathbf{r}),$$

$$\hat{G}(t_1, t_2; \mathbf{p}, \mathbf{r}) = \int d^2 r' e^{i\mathbf{p} \cdot \mathbf{r}'} \hat{G}(\mathbf{r}_1, t_1; \mathbf{r}_2, t_2 | \Phi), \tag{17}$$

where $\xi = \mathbf{p}^2 / 2m - \mu$, $\mathbf{r}' = \mathbf{r}_1 - \mathbf{r}_2$, and $\mathbf{r} = (\mathbf{r}_1 + \mathbf{r}_2)/2$. The electron polarization operators can be obtained in terms of Eqs. (13) and (17), i.e.,

$$\Pi^R(x_1, x_2) = \Pi^A(x_2, x_1) = \nu \int \frac{d\theta}{2\pi} \bigg[\delta(x_1 - x_2) + \frac{\pi \delta \operatorname{Tr}_s g^K(t_1, t_1; \mathbf{n}, \mathbf{r}_1)}{2 \delta \phi_1(\mathbf{r}_2, t_2)} \bigg],$$

$$\Pi^K(x_1, x_2) = \pi \nu \int \frac{d\theta}{2\pi} \frac{\delta \operatorname{Tr}_s [g^K(t_1, t_1; \mathbf{n}, \mathbf{r}_1) + g^Z(t_1, t_1; \mathbf{n}, \mathbf{r}_1)]}{2 \delta \phi_2(\mathbf{r}_2, t_2)}. \tag{18}$$

Since physical observables are determined by the Keldysh component of the quasiclassical Green's function, namely, $\langle g^K(t_1, t_2; \mathbf{n}, \mathbf{r}) \rangle_\Phi$ (here the subscript Φ refers that the functional average[29] is taken over the field Φ), we need to solve this component from the Eilenberger equation. Decomposing the Green's function in charge and spin components $\langle g^K(t_1, t_2; \mathbf{n}, \mathbf{r}) \rangle_\Phi = g_0^K + \mathbf{g}^K \cdot \vec{\sigma}$, one can obtain the charge and spin densities, respectively,

$$\rho(\mathbf{r}, t) = -\frac{1}{4} e\nu \int d\epsilon \langle g_0^K(t, \epsilon; \mathbf{n}, \mathbf{r}) \rangle_{\mathbf{n}},$$

$$\mathbf{S}(\mathbf{r}, t) = -\frac{1}{4} \nu \int d\epsilon \langle \mathbf{g}^K(t, \epsilon; \mathbf{n}, \mathbf{r}) \rangle_{\mathbf{n}}. \tag{19}$$

Now we turn to the kinetic equations for the two independent components g^K and g^Z. For $\langle g^Z \rangle_\Phi = 0$ in all orders of the perturbation theory, we have

$$g^K = \langle g^K \rangle_\Phi + \delta g^K, \quad g^Z = \delta g^Z,$$

where the fluctuation parts δg imply the effects contributed by the auxiliary bosonic fields. One can obtain from Eq. (14) that δg^Z obeys the following equation:

$$(\tilde{\partial}_t + \mathbf{v}_F \cdot \vec{\nabla}) \delta g^Z + i[\mathbf{b} \cdot \vec{\sigma}, \delta g^Z] - \frac{1}{\tau} [\delta g^Z - \langle \delta g^Z \rangle_{\mathbf{n}}]$$
$$= -2i \phi_2(\mathbf{r}, t_1) \delta(t_1 - t_2) I_s, \tag{20}$$

where I_s denotes the unit matrix in spin space, and we redefine the covariant derivative to be $\tilde{\partial}_t = \partial_{t1} + \partial_{t2}$. When deriving the above equation, we have used the conditions $g^R = \delta(t_1 - t_2) I_s - g^K \delta g^Z / 2$ and $g^A = -\delta(t_1 - t_2) I_s + \delta g^Z g^K / 2$. Equation (20) gives rise to

$$\delta g^Z(t_1, t_2; \mathbf{n}, \mathbf{r}) = 2i \delta(t_1 - t_2) \int d\mathbf{r}_1 dt_3 \int \frac{d\theta'}{2\pi}$$
$$\times \phi_2(\mathbf{r}_1, t_3) \Gamma_\rho(t_3 - t_1, \mathbf{n}', \mathbf{n}; \mathbf{r}_1, \mathbf{r}),$$

$$\Gamma_\rho(t,\mathbf{n}',\mathbf{n};\mathbf{r}_1,\mathbf{r}_2) = \int \frac{d\omega d^2\mathbf{q}}{(2\pi)^3} e^{i\mathbf{q}\cdot(\mathbf{r}_1-\mathbf{r}_2)-i\omega t}\Gamma_\rho(\mathbf{n}',\mathbf{n};\omega,\mathbf{q}),$$

$$(21)$$

where the diffusion propagator Γ_ρ is defined by

$$(-i\omega + iv_F\mathbf{n}\cdot\mathbf{q})\Gamma_\rho(\mathbf{n},\mathbf{n}';\omega,\mathbf{q}) + \frac{1}{\tau}[\Gamma_\rho(\mathbf{n},\mathbf{n}';\omega,\mathbf{q})$$

$$- \langle\Gamma_\rho(\mathbf{n},\mathbf{n}';\omega,\mathbf{q})\rangle_\mathbf{n}] = 2\pi\delta(\mathbf{n}-\mathbf{n}'). \quad (22)$$

After obtaining the explicit form of δg^Z, we can further solve the δg^K from the following relation:

$$(\tilde{\partial}_t + \mathbf{v_F}\cdot\vec{\nabla})\delta g^K + i[\mathbf{b}\cdot\vec{\sigma},\delta g^K] + \frac{1}{\tau}[\delta g^K - \langle\delta g^K\rangle_\mathbf{n}]$$

$$= 2i\phi_2(\mathbf{r},t_1)\delta(t_1-t_2)I_s - i[\phi_1(\mathbf{r},t_1) - \phi_1(\mathbf{r},t_2)]\langle g^K\rangle_\Phi$$

$$+ \frac{1}{4\tau}[\langle g^K\rangle_\Phi\langle\delta g^Z\langle g^K\rangle_\Phi\rangle_\mathbf{n} - \langle\langle g^K\rangle_\Phi\rangle_\mathbf{n}\delta g^Z\langle g^K\rangle_\Phi$$

$$- \langle g^K\rangle_\Phi\delta g^Z\langle\langle g^K\rangle_\Phi\rangle_\mathbf{n} + \langle\langle g^K\rangle_\Phi\delta g^Z\rangle_\mathbf{n}\langle g^K\rangle_\Phi]. \quad (23)$$

We take only the zeroth and first angular harmonics into account in the Keldysh component assumed spatial smoothness,

$$\langle g^K(t_1,t_2;\mathbf{n},\mathbf{r})\rangle_\Phi \approx \langle g^K(t_1,t_2;\mathbf{n}',\mathbf{r})\rangle_{\Phi,\mathbf{n}'}$$

$$+ 2\mathbf{n}\cdot\langle\mathbf{n}' g^K(t_1,t_2;\mathbf{n}',\mathbf{r})\rangle_{\Phi,\mathbf{n}'}. \quad (24)$$

Decomposing the fluctuating term in charge and spin components $\delta g^K = \delta g_0^K + \delta\mathbf{g}^K\cdot\vec{\sigma}$, one can easily obtain the explicit expression of the δg_0^K which is given in Eq. (A1). The fluctuation part $\delta\mathbf{g}^K$ related to the spin components fulfils the following equation,

$$(\tilde{\partial}_t + \mathbf{v}_F\cdot\vec{\nabla})\delta\mathbf{g}^K - 2\mathbf{b}\times\delta\mathbf{g}^K + \frac{1}{\tau}[\delta\mathbf{g}^K - \langle\delta\mathbf{g}^K\rangle_\mathbf{n}]$$

$$= -i[\phi_1(\mathbf{r},t_1) - \phi_1(\mathbf{r},t_2)](\langle\mathbf{g}^K(t_1,t_2;\mathbf{n},\mathbf{r})\rangle_{\Phi,\mathbf{n}}$$

$$+ 2\mathbf{n}\cdot\langle\mathbf{n}'\mathbf{g}^K(t_1,t_2;\mathbf{n}',\mathbf{r})\rangle_{\Phi,\mathbf{n}'}) + \frac{1}{2\tau}\{\langle g_0^K\rangle_{\Phi,\mathbf{n}}(\langle\delta g^Z\rangle_\mathbf{n}$$

$$- \delta g^Z)\langle\mathbf{g}^K\rangle_{\Phi,\mathbf{n}} + \langle\mathbf{g}^K\rangle_{\Phi,\mathbf{n}}(\langle\delta g^Z\rangle_\mathbf{n} - \delta g^Z)\langle g_0^K\rangle_{\Phi,\mathbf{n}}\}. \quad (25)$$

If denoting

$$Q = \begin{pmatrix} \delta g_x^K \\ \delta g_y^K \\ \delta g_z^K \end{pmatrix}, \quad L_k = \begin{pmatrix} g_x^K \\ g_y^K \\ g_z^K \end{pmatrix},$$

we can write Eq. (25) in the following matrix equation:

$$(\tilde{\partial}_t + \mathbf{v}_F\cdot\vec{\nabla})Q - 2\zeta Q + \frac{1}{\tau}(Q - \langle Q\rangle_\mathbf{n})$$

$$= -i[\phi_1(\mathbf{r},t_1) - \phi_1(\mathbf{r},t_2)](\langle L_k\rangle_{\Phi,\mathbf{n}} + 2\mathbf{n}\cdot\langle\mathbf{n}' L_k\rangle_{\Phi,\mathbf{n}'})$$

$$+ \frac{1}{2\tau}\{\langle g_0^K\rangle_{\Phi,\mathbf{n}}(\langle\delta g^Z\rangle_\mathbf{n} - \delta g^Z)\langle L_k\rangle_{\Phi,\mathbf{n}} + \langle L_k\rangle_{\Phi,\mathbf{n}}(\langle\delta g^Z\rangle_\mathbf{n}$$

$$- \delta g^Z)\langle g_0^K\rangle_{\Phi,\mathbf{n}}\}, \quad (26)$$

where the matrix ζ is given by

$$\zeta = \begin{pmatrix} 0 & 0 & -\alpha p_F\cos\theta \\ 0 & 0 & -\alpha p_F\sin\theta \\ \alpha p_F\cos\theta & \alpha p_F\sin\theta & 0 \end{pmatrix}.$$

Then Eq. (26) can be solved by utilizing the following expression:

$$(-i\omega + iv_F\mathbf{n}\cdot\mathbf{q} - 2\zeta)\Gamma_s(\mathbf{n},\mathbf{n}';\omega,\mathbf{q}) + \frac{1}{\tau}[\Gamma_s(\mathbf{n},\mathbf{n}';\omega,\mathbf{q})$$

$$- \langle\Gamma_s(\mathbf{n},\mathbf{n}';\omega,\mathbf{q})\rangle_\mathbf{n}] = 2\pi\delta(\mathbf{n}-\mathbf{n}'). \quad (27)$$

We give the explicit expression of $\delta\mathbf{g}^K$ in Eq. (A2) in Appendix A.

Since the concrete forms of δg^Z and δg^K have been obtained, we can write down the kinetic equation satisfied by the Keldysh function through averaging the K component of Eq. (14) over the auxiliary bosonic fields,

$$(\tilde{\partial}_t + \mathbf{v}_F\cdot\vec{\nabla})\langle g^K\rangle_\Phi + i[\mathbf{b}\cdot\vec{\sigma},\langle g^K\rangle_\Phi] = C_{\text{el}}\{\langle g^K\rangle_\Phi\} + C_{\text{in}}\{\langle g^K\rangle_\Phi\},$$

$$(28)$$

where the inelastic collision integral reads

$$C_{\text{in}}\{\langle g^K\rangle_\Phi\}(t_1,t_2;\mathbf{n},\mathbf{r}) = -i\langle[\phi_1(\mathbf{r},t_1) - \phi_1(\mathbf{r},t_2)]\delta g^K\rangle_\Phi,$$

$$(29)$$

and the elastic collision integral is given by

$$C_{\text{el}}\{\langle g^K\rangle_\Phi\}(t_1,t_2;\mathbf{n},\mathbf{r}) = \frac{1}{\tau}[\langle\langle g^K(t_1,t_2;\mathbf{n},r)\rangle_\Phi\rangle_\mathbf{n} - \langle g^K(t_1,t_2;\mathbf{n},\mathbf{r})\rangle_\Phi] + \int dt_3\frac{d\theta_1}{2\pi}[\langle g^K(t_1,t_3;\mathbf{n}_1,\mathbf{r})\rangle_\Phi\Lambda^A(t_3,t_2;\mathbf{n}_1,\mathbf{n};\mathbf{r})$$

$$- \langle g^K(t_1,t_3;\mathbf{n},\mathbf{r})\rangle_\Phi\Lambda^A(t_3,t_2;\mathbf{n},\mathbf{n}_1;\mathbf{r})] + \int dt_3\frac{d\theta_1}{2\pi}[\Lambda^R(t_1,t_3;\mathbf{n},\mathbf{n}_1;\mathbf{r})\langle g^K(t_3,t_2;\mathbf{n}_1,\mathbf{r})\rangle_\Phi$$

$$- \Lambda^R(t_1,t_3;\mathbf{n}_1,\mathbf{n};\mathbf{r})\langle g^K(t_3,t_2;\mathbf{n},\mathbf{r})\rangle_\Phi], \quad (30)$$

where

$$\Lambda^A(t_1, t_2; \mathbf{n}, \mathbf{n}_1; \mathbf{r}) = \frac{1}{4\tau} \int dt_3 \langle [\, \delta g^Z(t_1, t_3; \mathbf{n}_1, \mathbf{r})$$
$$- \delta g^Z(t_1, t_3; \mathbf{n}, \mathbf{r})] \delta g^K(t_3, t_2; \mathbf{n}, \mathbf{r}) \rangle_\Phi,$$

$$\Lambda^R(t_1, t_2; \mathbf{n}, \mathbf{n}_1; \mathbf{r}) = \frac{1}{4\tau} \int dt_3 \langle \delta g^K(t_1, t_3; \mathbf{n}, \mathbf{r}) [\, \delta g^Z(t_3, t_2; \mathbf{n}_1, \mathbf{r})$$
$$- \delta g^Z(t_3, t_2; \mathbf{n}, \mathbf{r})] \rangle_\Phi. \tag{31}$$

Substituting the explicit forms of δg^Z and δg^K given in Eqs. (21), (A1), and (A2) into Eq. (28), one gets the kinetic equation which can be used to study the influence of electron-electron interactions on the spin dynamics of 2DEGs with Rashba spin-orbit coupling. After some tedious calculation, we obtain the explicit expressions of the inelastic and elastic collision integrals, respectively, which are given in Eqs. (A3) and (A6) in Appendix A.

IV. SPIN DYNAMICS

After taking the average over the direction of the momentum $\mathbf{n}$, one can see from Eqs. (29) and (30) that the elastic collision integral vanishes,

$$\int \frac{d\theta}{2\pi} C_{\text{el}} \{ \langle g^K \rangle_\Phi \} (t, \epsilon; \mathbf{n}, \mathbf{r}) = 0, \tag{32}$$

but the average of the inelastic collision integral over the direction does not vanish. This means that the elastic collision integral preserves the number of electrons on a given energy shell defined by Eq. (A5), while the inelastic collision integral does not preserve it. When $t_1 = t_2$, Eq. (A4) gives rise to

$$\int d\epsilon\, C_{\text{in}} \{ \langle g^K \rangle_\Phi \} (t_1, \epsilon; \mathbf{n}, \mathbf{r}) = C_{\text{in}} \{ \langle g^K \rangle_\Phi \} (t_1, t_1; \mathbf{n}, \mathbf{r}).$$

One can see from Eq. (29) that the right-hand side is always zero. Thus we obtain

$$\int d\epsilon\, C_{\text{in}} \{ \langle g^K \rangle_\Phi \} (t, \epsilon; \mathbf{n}, \mathbf{r}) = 0. \tag{33}$$

This implies that not only the total number of electrons is conserved but also the number of electrons moving along a concrete direction $\mathbf{n}$ is conserved.

In decomposing the Green's function in charge and spin components in the approximation of Eq. (24), separating the zeroth and first angular harmonics, and utilizing Eqs. (32) and (33), we obtain from Eq. (28) that

$$v_F \mathbf{n} \cdot \vec{\nabla} \langle g^K(t, \epsilon; \mathbf{n}, \mathbf{r}) \rangle_{\Phi, \mathbf{n}} + i[\mathbf{b} \cdot \vec{\sigma}, \langle g^K(t, \epsilon; \mathbf{n}, \mathbf{r}) \rangle_{\Phi, \mathbf{n}}]$$
$$= C_{\text{el}} \{ \langle g^K \rangle_\Phi \} (t, \epsilon; \mathbf{n}, \mathbf{r}), \tag{34}$$

$$\partial_t \langle g^K(t, \epsilon; \mathbf{n}, \mathbf{r}) \rangle_{\Phi, \mathbf{n}} + i[\mathbf{b} \cdot \vec{\sigma}, 2\mathbf{n} \cdot \langle \mathbf{n}' g^K(t, \epsilon; \mathbf{n}', \mathbf{r}) \rangle_{\Phi, \mathbf{n}'}]$$
$$+ v_F \mathbf{n} \cdot \vec{\nabla} (2\mathbf{n} \cdot \langle \mathbf{n}' g^K(t, \epsilon; \mathbf{n}', \mathbf{r}) \rangle_{\Phi, \mathbf{n}'}) = 0. \tag{35}$$

There is no contribution of the inelastic collision integral to

the spin dynamics due to the condition (33). Solving Eq. (34) and substituting $\langle \mathbf{n}' g^K(t, \epsilon; \mathbf{n}', \mathbf{r}) \rangle_{\Phi, \mathbf{n}'}$ into Eq. (35), we can obtain the spin- and charge-diffusion equations. The diffusion equation for the charge density reads

$$\partial_t \rho - C_D \partial_{\mathbf{X}}^2 \rho = 0, \tag{36}$$

where $\partial_{\mathbf{X}} = (\partial_x, \partial_y)$ and $C_D = v_F^2 \tau / 2$. We introduce the distribution function f which reduces to the Fermi distribution in equilibrium,

$$f = f_0 + \vec{\sigma} \cdot \mathbf{f}_{\mathbf{k}} = \frac{1}{2} \left(1 - \frac{1}{2} g^K \right). \tag{37}$$

In the time τ, the charge density becomes isotropic but the spin-relaxation process does not start yet, hence[30]

$$g_0^K(\epsilon) = 2[1 - 2f_0(\epsilon)],$$

$$\mathbf{g}^K(t, \epsilon; \mathbf{r}) = -4\mathbf{f}_{\mathbf{k}}(t, \epsilon; \mathbf{r}), \tag{38}$$

where

$$f_0(\epsilon) = [f_+(\epsilon) + f_-(\epsilon)]/2,$$

$$\mathbf{f}_{\mathbf{k}}(t, \epsilon; \mathbf{r}) = [f_+(\epsilon) - f_-(\epsilon)]\mathbf{s}(t, \mathbf{r}),$$

$$f_\pm(\epsilon) = \left[\exp\left(\frac{\epsilon \mp \Delta\mu/2}{k_B T} \right) + 1 \right]^{-1}, \tag{39}$$

where $\mathbf{s} = (s_x, s_y, s_z)$ denotes the unit vector along the spin, $f_\pm(\epsilon)$ represents the distribution functions projected along the direction parallel or antiparallel to the unit vector $\mathbf{s}$ (all the energies are counted from the Fermi energy), and $\Delta\mu = (\mu_+ - \mu_-)$ refers to the difference between the chemical potentials $\mu_\pm$ of the electron-spin subsystems. The diffusion equations for the spin components are given by

$$\partial_t S_x - C_D \partial_{\mathbf{X}}^2 S_x - 2C_E \partial_x S_z + \frac{1}{\tau_s'} S_x = \frac{1}{\tau_{xx}^e} S_x + F_x(S_x, S_y, S_z),$$

$$\partial_t S_y - C_D \partial_{\mathbf{X}}^2 S_y - 2C_E \partial_y S_z + \frac{1}{\tau_s'} S_y = \frac{1}{\tau_{yy}^e} S_y + F_y(S_x, S_y, S_z),$$

$$\partial_t S_z - C_D \partial_{\mathbf{X}}^2 S_z + 2C_E \partial_x S_x + 2C_E \partial_y S_y + \frac{2}{\tau_s'} S_z$$

$$= \frac{1}{\tau_{zz}^e} S_z + F_z(S_x, S_y, S_z), \tag{40}$$

where $C_E = \alpha v_F p_F \tau$, $\tau_s' = 1/[2(\alpha p_F)^2 \tau]$ and $F_\ell(S_x, S_y, S_z)$ is a quadratic form of (S_x, S_y, S_z) lacking of the $S_\ell^2 (\ell = x, y, z)$ term. The characteristic times $\tau_{\ell\ell}^e$ describe the effect of the electron-electron interaction on the spin relaxation and their explicit expressions are given by

$$\frac{1}{\tau_{xx}^{e}} = \frac{2(\alpha p_F)^2 \tau}{M} \left\{ \int d\epsilon \int \frac{d\omega}{2\pi} \{[f_+(\epsilon-\omega) - f_-(\epsilon-\omega)] \right.$$
$$\left. \times \operatorname{Im}(R_2^{xx})g_0^K(\epsilon) + [f_+(\epsilon) - f_-(\epsilon)]R_1^{xx}g_0^K(\epsilon-\omega)\} \right\},$$

$$\frac{1}{\tau_{yy}^{e}} = \frac{2(\alpha p_F)^2 \tau}{M} \left\{ \int d\epsilon \int \frac{d\omega}{2\pi} \{[f_+(\epsilon-\omega) - f_-(\epsilon-\omega)] \right.$$
$$\left. \times \operatorname{Im}(R_2^{yy})g_0^K(\epsilon) + [f_+(\epsilon) - f_-(\epsilon)]R_1^{yy}g_0^K(\epsilon-\omega)\} \right\},$$

$$\frac{1}{\tau_{zz}^{e}} = \frac{1}{\tau_{xx}^{e}} + \frac{1}{\tau_{yy}^{e}}$$
$$= \frac{2(\alpha p_F)^2 \tau}{M} \left\{ \int d\epsilon \int \frac{d\omega}{2\pi} \{[f_+(\epsilon-\omega) - f_-(\epsilon-\omega)] \right.$$
$$\times [\operatorname{Im}(R_2^{xx}) + \operatorname{Im}(R_2^{yy})]g_0^K(\epsilon) + [f_+(\epsilon) - f_-(\epsilon)]$$
$$\left. \times (R_1^{xx} + R_1^{yy})g_0^K(\epsilon-\omega)\} \right\}, \qquad (41)$$

where $M = \int d\epsilon (f_+(\epsilon) - f_-(\epsilon))$. In order to obtain the concrete expressions of the characteristic times $\tau_{\ell\ell}^{e}$, we first take the energy integration in Eq. (41). Since the spin splitting is small, i.e.,

$$|\mu_+ - \mu_-| \ll |\mu_+|, |\mu_-|,$$

the energy integration can be taken as follows:

$$\frac{1}{M} \int_{-\infty}^{\infty} d\epsilon (f_+(\epsilon-\omega) - f_-(\epsilon-\omega))(f_+(\epsilon) + f_-(\epsilon))$$

$$\approx \frac{2 \int_{-\infty}^{\infty} d\epsilon \frac{\partial f_0(\epsilon-\omega)}{\partial \epsilon} f_0(\epsilon)}{\int_{-\infty}^{\infty} d\epsilon \frac{\partial f_0(\epsilon)}{\partial \epsilon}} = 1 - \frac{\partial}{\partial \omega} \left(\omega \coth \frac{\omega}{2k_B T} \right).$$

$$(42)$$

After the energy integration, the characteristic times $\tau_{\ell\ell}^{e}$ have the forms

$$\frac{1}{\tau_{xx}^{e}} = \frac{1}{\tau_{yy}^{e}} = 8(\alpha p_F)^2 \tau \int_0^{\infty} \frac{d\omega}{2\pi} \left[\frac{\partial}{\partial \omega} \left(\omega \coth \frac{\omega}{2k_B T} \right) \right]$$
$$\times [\operatorname{Im}(R_2^{xx}) - R_1^{xx}]. \qquad (43)$$

The detail of the calculations of the kernels R_1^{ij} and $\operatorname{Im} R_2^{ij}$ are given in Appendix B. Now we discuss the influence of the electron-electron interaction on the spin-relaxation time in the ballistic regime $T\tau \gg 1$.

We can obtain the characteristic time τ_{xx}^{e} in the ballistic regime utilizing the kernels R_1^{ij} and $\operatorname{Im} R_2^{ij}$ in Eq. (B7),

$$\frac{1}{\tau_{xx}^{e}}(T\tau \gg 1) = 8(\alpha p_F)^2 \tau \int_0^{\infty} \frac{d\omega}{2\pi} \left[\frac{\partial}{\partial \omega} \left(\omega \coth \frac{\omega}{2k_B T} \right) \right]$$
$$\times \left[\frac{-1}{4\pi\nu v_F^2} \left(\frac{3\pi}{2} + \tan^{-1}\omega\tau - \frac{2\omega\tau}{1 + \omega^2\tau^2} \right) \right]$$
$$\approx \frac{-4(\alpha p_F)^2 \tau}{\nu v_F^2} \int_0^{\infty} \frac{d\omega}{2\pi} \left[\frac{\partial}{\partial \omega} \left(\omega \coth \frac{\omega}{2k_B T} \right) \right]$$
$$= \frac{2(\alpha p_F)^2 \tau}{\pi \nu v_F^2} \left(2k_B T - E_F \coth \frac{E_F}{2k_B T} \right), \qquad (44)$$

where $\tan^{-1}(\omega\tau)$ is replaced by $\pi/2$ for $\omega\tau \gg 1$ in the ballistic regime and E_F is in the place of the upper limit of the integral. In the low-temperature regime $k_B T \ll E_F$, the second term approaches a constant independent of the temperature, so the first term manifests the temperature effect in the contribution of the electron-electron interaction to the spin-relaxation time.

When the total spin density **S** is spatially homogeneous and parallel to the ℓth axis of the coordinate frame, the contribution of $F_\ell(S_x, S_y, S_z)$ vanishes, namely, $F_\ell(S_x, S_y, S_z) = 0$. The diffusion equations for spin components S_ℓ can be simplified, for example,

$$\partial_t S_x = -\frac{1}{\tau_s'} S_x + \frac{1}{\tau_{xx}^{e}} S_x = -\frac{1}{\tau_{xx}^{s}} S_x, \qquad (45)$$

where $\tau_{xx}^{s} = \tau_s'/(1 - \tau_s'/\tau_{xx}^{e})$. Therefore, the spin-relaxation times can be determined by τ_s' and $\tau_{\ell\ell}^{e}$, consequently,

$$\tau_{xx}^{s} = \frac{\tau_s'}{1 - \dfrac{\tau_s'}{\tau_{xx}^{e}}}, \quad \tau_{yy}^{s} = \frac{\tau_s'}{1 - \dfrac{\tau_s'}{\tau_{yy}^{e}}},$$

$$(\tau_{zz}^{s})^{-1} = (\tau_{xx}^{s})^{-1} + (\tau_{yy}^{s})^{-1}. \qquad (46)$$

We can see that the total spin decays exponentially when $0 < \tau_s'/\tau_{\ell\ell}^{e} < 1$. In terms of the explicit forms of the characteristic times $\tau_{\ell\ell}^{e}$ in the ballistic regime, the spin-relaxation times involving the effect of the electron-electron interaction take the following forms:

$$\tau_{xx}^{s} = \tau_{yy}^{s} = 2\tau_{zz}^{s} = \frac{\tau_s'}{1 - \left(\dfrac{T}{T_F} - \dfrac{1}{2} \right)}, \quad T\tau \gg 1, \qquad (47)$$

where $T_F = E_F/k_B$ is the Fermi temperature. It is worthwhile to indicate that there exists an obvious enhancement of the spin-relaxation time with increment of the temperature in the ballistic regime. The increasing amplitude of the spin-relaxation time depends on the ratio of the temperature to the Fermi temperature. In conclusion, an obvious enhancement of the spin-relaxation time can be induced by the electron-electron interaction in the ballistic regime for systems under consideration.

V. SUMMARY

In the above, we presented a theoretical study of the influence of electron-electron interactions on the spin dynam-

ics for 2DEGs with Rashba spin-orbit coupling. We employed the path-integral approach and the quasiclassical Green's function to deal with the electron-electron interaction. With the help of the auxiliary Bose field, the electron-electron interaction was decoupled via the Hubbard-Stratonovich transformation. Then one is able to derive the Eilenberger equation by using the Green's function after the transformation. Through tedious calculation, we further derived the spin- and charge-diffusion equations from which the spin-relaxation time can be given explicitly. We analyzed the influence of the electron-electron interaction on the spin-relaxation time in the ballistic regime and found an obvious enhancement of the spin-relaxation time with the increment of the temperature T. The increasing amplitude of the spin-relaxation time depends on the ratio of the temperature to the Fermi temperature. The electron-electron interaction changes the wave vector **k** and hence results in the variation of the spin precession vector. This exhibits that the electron-

electron interaction plays an important role in the spin relaxation of electrons when the D'yakonov-Perel spin-relaxation mechanism dominates. It is expected to be helpful in understanding the spin dynamics of 2DEGs with spin-orbit couplings and electron-electron interactions. Our formulation can also extend to the case of bulk inversion asymmetry, namely, the additional Dresselhaus term[31] with $\mathbf{b} = \beta(p_x, -p_y) + \gamma(p_x p_y^2 - p_y p_x^2)$.

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APPENDIX A: EXPLICIT FORMS

The explicit form of δg_0^K is

$$\delta g_0^K(t_1, t_2; \mathbf{n}, \mathbf{r}) = -i \int dt_\theta \, d^2 r_1 [\phi_1(\mathbf{r}_1, t_1 - t_\theta) - \phi_1(\mathbf{r}_1, t_2 - t_\theta)] \int \frac{d\theta'}{2\pi} \Gamma_\rho(t_\theta, \mathbf{n}, \mathbf{n}'; \mathbf{r}, \mathbf{r}_1) \times \{\langle g_0^K(t_1 - t_\theta, t_2 - t_\theta; \mathbf{n}_1, \mathbf{r})\rangle_{\Phi, \mathbf{n}_1}$$

$$+ 2\mathbf{n}' \cdot \langle \mathbf{n}_1 g_0^K(t_1 - t_\theta, t_2 - t_\theta; \mathbf{n}_1, \mathbf{r})\rangle_{\Phi, \mathbf{n}_1}\} + \int \frac{d\theta'}{2\pi} \frac{d\theta''}{2\pi} \int d^2 r_1 dt_\theta \Gamma_\rho(t_\theta, \mathbf{n}, \mathbf{n}'; \mathbf{r}, \mathbf{r}_1) \bigg\{ 2i\phi_2(\mathbf{r}_1, t_1 - t_\theta) \delta(t_1 - t_2)$$

$$+ \frac{i}{\tau} [\langle \Gamma_\rho(t_4 - t_3, \mathbf{n}'', \mathbf{n}_1; \mathbf{r}_2, \mathbf{r}_1)\rangle_{\mathbf{n}_1} - \Gamma_\rho(t_4 - t_3, \mathbf{n}'', \mathbf{n}; \mathbf{r}_2, \mathbf{r}_1)] \phi_2(\mathbf{r}_2, t_4) \times [\langle g_0^K(t_1 - t_\theta, t_3; \mathbf{n}_1, \mathbf{r})\rangle_{\Phi, \mathbf{n}_1}$$

$$\times \langle g_0^K(t_3, t_2 - t_\theta; \mathbf{n}_1, \mathbf{r})\rangle_{\Phi, \mathbf{n}_1} + \langle \mathbf{g}_K(t_1 - t_\theta, t_3; \mathbf{n}_1, \mathbf{r})\rangle_{\Phi, \mathbf{n}_1} \cdot \langle \mathbf{g}_K(t_3, t_2 - t_\theta; \mathbf{n}_1, \mathbf{r})\rangle_{\Phi, \mathbf{n}_1}] \bigg\}. \tag{A1}$$

The explicit expression of $\delta \mathbf{g}^K$ is

$$Q(t_1, t_2; \mathbf{n}, \mathbf{r}) = -i \int dt_\theta \, d^2 r_1 [\phi_1(\mathbf{r}_1, t_1 - t_\theta) - \phi_1(\mathbf{r}_1, t_2 - t_\theta)] \int \frac{d\theta'}{2\pi} \Gamma_s(t_\theta, \mathbf{n}, \mathbf{n}'; \mathbf{r}, \mathbf{r}_1) \times \{\langle L_k(t_1 - t_\theta, t_2 - t_\theta; \mathbf{n}_1, \mathbf{r})\rangle_{\Phi, \mathbf{n}_1}$$

$$+ 2\mathbf{n}' \cdot \langle \mathbf{n}_1 L_k(t_1 - t_\theta, t_2 - t_\theta; \mathbf{n}_1, \mathbf{r})\rangle_{\Phi, \mathbf{n}_1}\} + \frac{2i}{\tau} \int \frac{d\theta'}{2\pi} \frac{d\theta''}{2\pi} \int d^2 r_1 dt_\theta \Gamma_s(t_\theta, \mathbf{n}, \mathbf{n}'; \mathbf{r}, \mathbf{r}_1) \langle g_0^K(t_1 - t_\theta, t_3; \mathbf{n}_1, \mathbf{r})\rangle_{\Phi, \mathbf{n}_1}$$

$$\times \{[\langle \Gamma_\rho(t_4 - t_3, \mathbf{n}'', \mathbf{n}_1; \mathbf{r}_2, \mathbf{r}_1)\rangle_{\mathbf{n}_1} - \Gamma_\rho(t_4 - t_3, \mathbf{n}'', \mathbf{n}_1; \mathbf{r}_2, \mathbf{r}_1)] \times \phi_2(\mathbf{r}_4, t_2) \langle L_k(t_3, t_2 - t_\theta; \mathbf{n}_1, \mathbf{r})\rangle_{\Phi, \mathbf{n}_1}\}. \tag{A2}$$

The inelastic collision integral reads

$$C_{\text{in}}\{\langle g^K\rangle_\Phi\}(t, \epsilon; \mathbf{n}, \mathbf{r}) = \frac{i}{2\tau} \int d^2 r_1 d^2 r_2 \int \frac{d\omega}{2\pi} [D^R(\omega; \mathbf{r}, \mathbf{r}_2) - D^A(\omega; \mathbf{r}_2, \mathbf{r})] \times [\langle \Gamma_\rho(\omega; \mathbf{r}_2, \mathbf{r}_1)\rangle\langle \Gamma_\rho(-\omega; \mathbf{r}, \mathbf{r}_1)\rangle$$

$$- \langle \Gamma_\rho(-\omega; \mathbf{r}, \mathbf{r}_1) \Gamma_\rho(\omega; \mathbf{r}_2, \mathbf{r}_1)\rangle][\langle g_0^K(t, \epsilon - \omega; \mathbf{n}, \mathbf{r})\rangle_{\Phi, \mathbf{n}} \langle g_0^K(t, \epsilon; \mathbf{n}, \mathbf{r})\rangle_{\Phi, \mathbf{n}}$$

$$+ \langle \mathbf{g}^K(t, \epsilon - \omega; \mathbf{n}, \mathbf{r})\rangle_{\Phi, \mathbf{n}} \cdot \langle \mathbf{g}^K(t, \epsilon; \mathbf{n}, \mathbf{r})\rangle_{\Phi, \mathbf{n}}] + \frac{i}{\tau} \int d^2 r_1 d^2 r_2 \int \frac{d\omega}{2\pi} [D^R(\omega; \mathbf{r}, \mathbf{r}_2) - D^A(\omega; \mathbf{r}_2, \mathbf{r})] \sigma_m$$

$$\times [\langle \Gamma_s(-\omega; \mathbf{r}_2, \mathbf{r}_1)\rangle\langle \Gamma_\rho(-\omega; \mathbf{r}, \mathbf{r}_1)\rangle - \langle \Gamma_s(-\omega; \mathbf{r}, \mathbf{r}_1) \Gamma_\rho(\omega; \mathbf{r}_2, \mathbf{r}_1)\rangle] \langle g_0^K(t, \epsilon - \omega; \mathbf{n}, \mathbf{r})\rangle_{\Phi, \mathbf{n}} \langle L_k(t, \epsilon; \mathbf{n}, \mathbf{r})\rangle_{\Phi, \mathbf{n}}, \tag{A3}$$

where

$$C_{\text{in}}\{\langle g^K\rangle_\Phi\}(t_1, t_2; \mathbf{n}, \mathbf{r}) = \int \frac{d\epsilon}{2\pi} C_{\text{in}}\{\langle g^K\rangle_\Phi\}\left(\frac{t_1 + t_2}{2}, \epsilon; \mathbf{n}, \mathbf{r}\right) e^{i\epsilon(t_2 - t_1)}, \tag{A4}$$

and $\langle \Gamma_{\rho(s)}\rangle$ means angular averaging defined in Eq. (A10), the matrix $\sigma_m = (\sigma_x, \sigma_y, \sigma_z)$ and the temporal transformation of the Green's function have been used due to a much faster dependence on the difference $t_1 - t_2$ than on the $t_1 + t_2$,

$$g^K(t_1, t_2; \mathbf{n}, \mathbf{r}) = \int \frac{d\epsilon}{2\pi} g^K\left(\frac{t_1 + t_2}{2}, \epsilon; \mathbf{n}, \mathbf{r}\right) e^{i\epsilon(t_2 - t_1)}, \tag{A5}$$

the propagators of auxiliary fields have the same transformation. The elastic collision integral can be written as

$$C_{\text{el}}\{\langle g^K \rangle_\Phi\}(t, \epsilon; \mathbf{n}, \mathbf{r}) = -\frac{2}{\tau} \mathbf{n}_\iota \langle \mathbf{n}_\iota g^K(t, \epsilon; \mathbf{n}, \mathbf{r}) \rangle_{\Phi, \mathbf{n}} - \frac{2}{\tau} \int \frac{d\omega}{2\pi} \mathbf{n}_\iota R_1^{\iota J}(\omega) \langle g_0^K(t, \epsilon - \omega; \mathbf{n}, \mathbf{r}) \rangle_{\Phi, \mathbf{n}} \times \langle \mathbf{n}_J g^K(t, \epsilon; \mathbf{n}, \mathbf{r}) \rangle_{\Phi, \mathbf{n}} + \frac{i}{\tau} \int \frac{d\omega}{2\pi} \mathbf{n}_\iota R_2^{\iota J}(\omega)$$

$$\times \langle \mathbf{n}_J g^K(t, \epsilon - \omega; \mathbf{n}, \mathbf{r}) \rangle_{\Phi, \mathbf{n}} \langle g^K(t, \epsilon; \mathbf{n}, \mathbf{r}) \rangle_{\Phi, \mathbf{n}} - \frac{i}{\tau} \int \frac{d\omega}{2\pi} \mathbf{n}_\iota [R_2^{\iota J}(\omega)]^\star \langle g^K(t, \epsilon; \mathbf{n}, \mathbf{r}) \rangle_{\Phi, \mathbf{n}} \langle \mathbf{n}_J g^K(t, \epsilon - \omega; \mathbf{n}, \mathbf{r}) \rangle_{\Phi, \mathbf{n}}$$

$$+ \frac{i}{\tau} \int \frac{d\omega}{2\pi} \mathbf{n}_\iota \{ \sigma_m R_3^{\iota J}(\omega) \langle L_k(t, \epsilon - \omega; \mathbf{n}, \mathbf{r}) \rangle_{\Phi, \mathbf{n}} \langle \mathbf{n}_J g^K(t, \epsilon; \mathbf{n}, \mathbf{r}) \rangle_{\Phi, \mathbf{n}} - \langle \mathbf{n}_J g^K(t, \epsilon; \mathbf{n}, \mathbf{r}) \rangle_{\Phi, \mathbf{n}} \sigma_m [R_3^{\iota J}(\omega)]^\star$$

$$\times \langle L_k(t, \epsilon - \omega; \mathbf{n}, \mathbf{r}) \rangle_{\Phi, \mathbf{n}} \}, \tag{A6}$$

where $\mathbf{n}_{\iota(J)}$ refers to the $\iota(J)$ component of the unit vector $\mathbf{n}$, with $\iota, J = x, y$ and the kernels $R_1^{\iota J}(\omega) - R_3^{\iota J}(\omega)$ in Eq. (A6) are defined by

$$R_1^{\iota J}(\omega) = \text{Im} \int \frac{d^2 q}{(2\pi)^2} D^R(\omega; \mathbf{q}) \left\{ \langle \Gamma_\rho(\mathbf{n}, \omega; \mathbf{q}) \mathbf{n}_J \rangle \langle \mathbf{n}_\iota \Gamma_\rho \rangle \right.$$
$$\left. - \frac{1}{2} \delta_{\iota J} (\langle \Gamma_\rho \rangle \langle \Gamma_\rho \rangle - \langle \Gamma_\rho \Gamma_\rho \rangle) \right\}, \tag{A7}$$

$$R_2^{\iota J}(\omega) = \int \frac{d^2 q}{(2\pi)^2} D^R(\omega; \mathbf{q}) \{ \langle \Gamma_\rho \rangle \langle \mathbf{n}_\iota \Gamma_\rho \mathbf{n}_J \rangle - \langle \Gamma_\rho \mathbf{n}_\iota \Gamma_\rho \mathbf{n}_J \rangle$$
$$- \langle \Gamma_\rho \mathbf{n}_\iota \rangle \langle \Gamma_\rho \mathbf{n}_J \rangle \}, \tag{A8}$$

$$R_3^{\iota J}(\omega) = \int \frac{d^2 q}{(2\pi)^2} D^R(\omega; \mathbf{q}) \left\{ \langle \Gamma_\rho \mathbf{n}_J \rangle \langle \mathbf{n}_\iota \Gamma_s \rangle - \frac{1}{2} \delta_{\iota J} (\langle \Gamma_\rho \rangle \langle \Gamma_s \rangle \right.$$
$$\left. - \langle \Gamma_\rho \Gamma_s \rangle) \right\}, \tag{A9}$$

where we have introduced the notation

$$\langle f \Gamma_{\rho(s)} h \rangle = \int \frac{d\theta d\theta'}{(2\pi)^2} f(\mathbf{n}) \Gamma_{\rho(s)}(\mathbf{n}, \mathbf{n}'; \omega, \mathbf{q}) h(\mathbf{n}'). \tag{A10}$$

APPENDIX B: CALCULATION OF THE KERNELS $R_i^{\iota J}$

According to the definition of the diffusion propagator Γ_ρ in Eq. (22), we can obtain

$$\Gamma_\rho(\mathbf{n}, \mathbf{n}'; \omega, \mathbf{q}) = 2\pi \delta(\mathbf{n} - \mathbf{n}') \Gamma_{\rho 0}(\mathbf{n}; \omega, \mathbf{q})$$
$$+ \Gamma_{\rho 0}(\mathbf{n}; \omega, \mathbf{q}) \Gamma_{\rho 0}(\mathbf{n}'; \omega, \mathbf{q}) \frac{1}{\tau - \frac{1}{\Upsilon}}, \tag{B1}$$

where

$$\Gamma_{\rho 0}(\mathbf{n}; \omega, \mathbf{q}) = \frac{1}{-i\omega + iv_F \mathbf{n} \cdot \mathbf{q} + \frac{1}{\tau}}$$

$$= \frac{1}{-i\omega + iv_F q \cos(\phi - \phi_q) + \frac{1}{\tau}},$$

$$\Upsilon = \sqrt{\left(-i\omega + \frac{1}{\tau}\right)^2 + v_F^2 q^2}, \tag{B2}$$

with ϕ_q being the angle between the wave vector $\mathbf{q}$ and the x axis. In terms of the explicit form of the diffusion propagator, one can obtain

$$\langle \Gamma_\rho \rangle = \frac{\tau}{\Upsilon \tau - 1},$$

$$\langle \Gamma_\rho n_x \rangle = \langle n_x \Gamma_\rho \rangle = \frac{\tau}{iv_F q(\Upsilon \tau - 1)} \left(\Upsilon + i\omega - \frac{1}{\tau} \right) \cos \phi_q,$$

$$\langle \Gamma_\rho n_y \rangle = \langle n_y \Gamma_\rho \rangle = \frac{\tau}{iv_F q(\Upsilon \tau - 1)} \left(\Upsilon + i\omega - \frac{1}{\tau} \right) \sin \phi_q,$$

$$\langle \Gamma_\rho \Gamma_\rho \rangle = \frac{-i\omega + \frac{1}{\tau}}{\Upsilon \left(\Upsilon - \frac{1}{\tau} \right)^2},$$

$$\langle \Gamma_\rho n_x \Gamma_\rho \rangle = \frac{1}{\Upsilon} \sin^2 \phi_q - \frac{\Upsilon}{v_F^2 q^2 (\Upsilon \tau - 1)}$$
$$\times \left(1 - \frac{-i\omega + \frac{1}{\tau}}{\Upsilon} \right)^2 \cos^2 \phi_q,$$

$$\langle \Gamma_\rho n_y \Gamma_\rho \rangle = \frac{1}{\Upsilon} \cos^2 \phi_q - \frac{\Upsilon}{v_F^2 q^2 (\Upsilon \tau - 1)}$$

$$\times \left(1 - \frac{-i\omega + \dfrac{1}{\tau}}{\Upsilon} \right)^2 \sin^2 \phi_q,$$

$$\langle \Gamma_\rho n_x \Gamma_\rho n_x \rangle = \frac{\tau}{\Upsilon \tau - 1}$$

$$\times \left(\frac{-i\omega + \dfrac{1}{\tau}}{\Upsilon^2} \sin^2 \phi_q - \frac{\Upsilon - \left(i\omega + \dfrac{1}{\tau} \right)}{\Upsilon^2 (\Upsilon \tau - 1)} \cos^2 \phi_q \right),$$

$$\langle \Gamma_\rho n_y \Gamma_\rho n_y \rangle = \frac{\tau}{\Upsilon \tau - 1}$$

$$\times \left(\frac{-i\omega + \dfrac{1}{\tau}}{\Upsilon^2} \cos^2 \phi_q - \frac{\Upsilon - \left(i\omega + \dfrac{1}{\tau} \right)}{\Upsilon^2 (\Upsilon \tau - 1)} \sin^2 \phi_q \right).$$

$$(B3)$$

Utilizing the above formulas, we find that the kernels $R_1^{IJ}(\omega)$ and $R_2^{IJ}(\omega)$ are diagonal, $R_i^{IJ} = \delta_{IJ} R_i$, which can be written as

$$R_1(\omega) = - \operatorname{Im} \int_0^\infty \frac{q dq}{4\pi} D^R(\omega;q) \left\{ \frac{1}{v_F^2 q^2} \left(\frac{\Upsilon + i\omega - \dfrac{1}{\tau}}{\Upsilon - \dfrac{1}{\tau}} \right)^2 \right.$$

$$\left. + \frac{\Upsilon + i\omega - \dfrac{1}{\tau}}{\Upsilon \left(\Upsilon - \dfrac{1}{\tau} \right)^2} \right\},$$

$$\operatorname{Im} R_2(\omega) = \operatorname{Im} \int_0^\infty \frac{q dq}{4\pi} D^R(\omega;q) \left\{ \frac{1}{v_F^2 q^2} \frac{\left[\Upsilon + i\omega - \dfrac{1}{\tau} \right]^2}{\Upsilon \left(\Upsilon - \dfrac{1}{\tau} \right)} \right.$$

$$\left. + \frac{\Upsilon + i\omega - \dfrac{1}{\tau}}{\Upsilon \left(\Upsilon - \dfrac{1}{\tau} \right)^2} \right\}.$$

$$(B4)$$

It is not difficult to calculate the concrete forms of the electron polarization operators from Eq. (18), for example,

$$\Pi^R(\omega,q) = \nu \left[1 + \frac{i\omega}{\Upsilon - \dfrac{1}{\tau}} \right]. \tag{B5}$$

Substituting the polarization operator into Eq. (12), we obtain the propagator of the Bose fields, i.e.,

$$D^R(\omega,q) = \frac{D_0^R}{1 - D_0^R \Pi^R} = \frac{-2\pi e^2/q}{1 + \dfrac{2\pi e^2}{q} \nu \left[1 + \dfrac{i\omega}{\Upsilon - \dfrac{1}{\tau}} \right]} \approx - \frac{1}{\Pi^R}$$

$$= - \frac{1}{\nu} \frac{\Upsilon - \dfrac{1}{\tau}}{\Upsilon - \dfrac{1}{\tau} + i\omega}, \tag{B6}$$

where the approximation in the second line corresponds to the unitary limit associating with larger distances than the screening radius.

We obtain the concrete expressions of the kernels R_1 and R_2 in the ballistic regime $T\tau \gg 1$,

$$R_1(T\tau \gg 1) \propto \frac{1}{8\pi \nu v_F^2} \left(\frac{3\pi}{2} + \tan^{-1} \omega\tau - \frac{2\omega\tau}{1 + \omega^2 \tau^2} \right),$$

$$\operatorname{Im} R_2(T\tau \gg 1) \propto - \frac{1}{8\pi \nu v_F^2} \left(\frac{3\pi}{2} + \tan^{-1} \omega\tau - \frac{2\omega\tau}{1 + \omega^2 \tau^2} \right). \tag{B7}$$

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